The effects of topography on mixing rates in controlled exchange flows

by Tjipto Prastowo

The effects of topography on mixing rates in controlled exchange flows

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ABSTRACT: We examine irreversible mixing in buoyancy-driven baroclinic exchange flows past bottom topography. In all cases the shear between the counter-flowing layers is unstable in the vicinity of a hydraulic control situated above the topography. The overall amount of irreversible turbulent mixing produced by overturning billows during a given time is determined from measurements of vertical density profiles after the exchange. The potential energy changes are used to evaluate the global mixing efficiency, defined as the fraction of the available potential energy released into kinetic energy that is converted to potential energy through irreversible mixing. In all cases examined, the mixing efficiency is no more than 11% ± 1%, which corresponds to the maximum mixing efficiency found in previous work for exchange flows through a lateral contraction. The mixing efficiency is found to reduce as the fractional depth of the topography increases and as the topographic slope decreases.

1 INTRODUCTION

The exchange of fluid through a constriction or past a sill that connects reservoirs of different density is generally unstable to shear. Strong turbulence and mixing occurs between the counter-flowing layers. These phenomena are neglected in hydraulic theories used to model such flows (Lawrence 1990). However, it has been shown that the mixing both alters the range of densities in the flow and reduces the exchange transport (Helfrich 1995, Gregg & Öszoy 2002, Winters & Siem 2000) and this mixing may contribute significantly to the vertical mixing in the deep oceans (Bryden & Nurser 2003). Recent laboratory studies (Prastowo et al. 2008, 2009) have quantified these effects for baroclinic exchange flows past a variety of lateral constrictions and bottom sills. Their important findings were that the overall mixing efficiency characterising the shear-generated mixing is substantially less than the value of 20% often assumed for stratified turbulence, and that the exchange transport is 70–80% of that predicted by inviscid hydraulic theory.

Our aims in this paper are twofold. Firstly we report on new experiments undertaken to investigate the role of sill height and slope on mixing in a baroclinic exchange flow. Secondly we examine these results in the context of our previous work, with the aim of building a more complete picture of the effects that sill and constriction geometry has on the mixing in exchange flows.

2 METHODS

A brief overview of the experimental apparatus and measurements is given here; the reader is referred to Prastowo et al. (2008, 2009) for a detailed description. All experiments were con-

Table 1. Summary of variables used in experiments. Experiments in the "J" and "K" series examined the effect of density difference for a given sill geometry, and experiments in the "L" series explored the effect of sill geometry for generally large density differences. Note that for all experiments the constant-width portion of the lateral constriction in which the sill resides was of length $L_c = 0.5$m and width $b_0 = 0.06$ m.

Experiment	Sill height h (m)	Sill length L (m)	Water depth H (m)	Density difference $\Delta\rho/\rho_0$
J1	0.1	0	0.2	0.007
J2	0.1	0	0.2	0.048
J3	0.1	0	0.2	0.03
J4	0.1	0	0.2	0.019
J5	0.1	0	0.2	0.067
J6	0.1	0	0.2	0.094
K1	0.1	0.25	0.2	0.007
K2	0.1	0.25	0.2	0.049
K3	0.1	0.25	0.2	0.029
K4, K5	0.1	0.25	0.2	0.019
K6	0.1	0.25	0.2	0.067
K7	0.1	0.25	0.2	0.095
K8	0.1	0.25	0.2	0.079
K9	0.1	0.25	0.2	0.003
L1, L2	0.06	0.25	0.2	0.05
L3, L4	0.14	0.25	0.2	0.05
L5	0.14	0.25	0.2	0.019
L6	0.14	0.25	0.2	0.08
L8, L9	0.14	0.25	0.175	0.05
L13	0.06	0.25	0.12	0.05
L14	0.06	0.25	0.09	0.05
L15	0.14	0.25	0.28	0.05
L18	0.117	0.486	0.234	0.051
L19	0.117	0.486	0.234	0.095
L20	0.117	0.486	0.15	0.05

ducted in a channel of total length 5.26 m and width 0.2 m with a smooth-sided lateral constriction and a bottom sill placed at its midpoint. In all cases, the bottom sill spanned the minimum width (b_0 = 0.06 m) of the channel with a vertical cross-section that was uniform. The channel was initially separated into left and right reservoirs by a vertical barrier inserted at the midpoint. A density difference was created across the barrier by adding salt to one reservoir, and the relative free-surface height of the reservoirs adjusted such that the ensuing exchange flow was purely baroclinic.

In the series of experiments reported here a number of variables were altered (see Table 1), including the sill height and length (defined as the length of the down-slope portion of the sill along which water from the dense reservoir flows), the total water depth and the density difference across the sill. The vertical cross-section of the bottom sill was usually triangular, but a vertical weir (denoted by a sill length L = 0 m) was used in some experiments. The important dimensionless parameters are the sill height normalised by the total depth h/H, the sill slope h/L and a Reynolds number (calculated using a velocity scale based on the density difference between reservoirs).

The free-surface heights and density of each reservoir were accurately measured before each experiment. The vertical barrier was rapidly withdrawn to initiate the exchange flow, and was replaced just prior to the time when the density currents from the exchange impinged on the end walls of the channel. The fluid in the channel was then allowed to come to rest, before the density profiles and free-surface heights in each reservoir were accurately measured. The decrease of the potential energy of the system can then be calculated directly (Prastowo et al. 2008, 2009). Conservation of salt is further used to calculate the potential energy of a hypothetical flow in which the exchange of salt is the same, but in which no mixing occurs. This hypothetical state is thus used to calculate both the release of (available) potential energy from the system ΔAPE (to the flow) and the potential energy increase due to resultant irreversible mixing ΔPE_m.

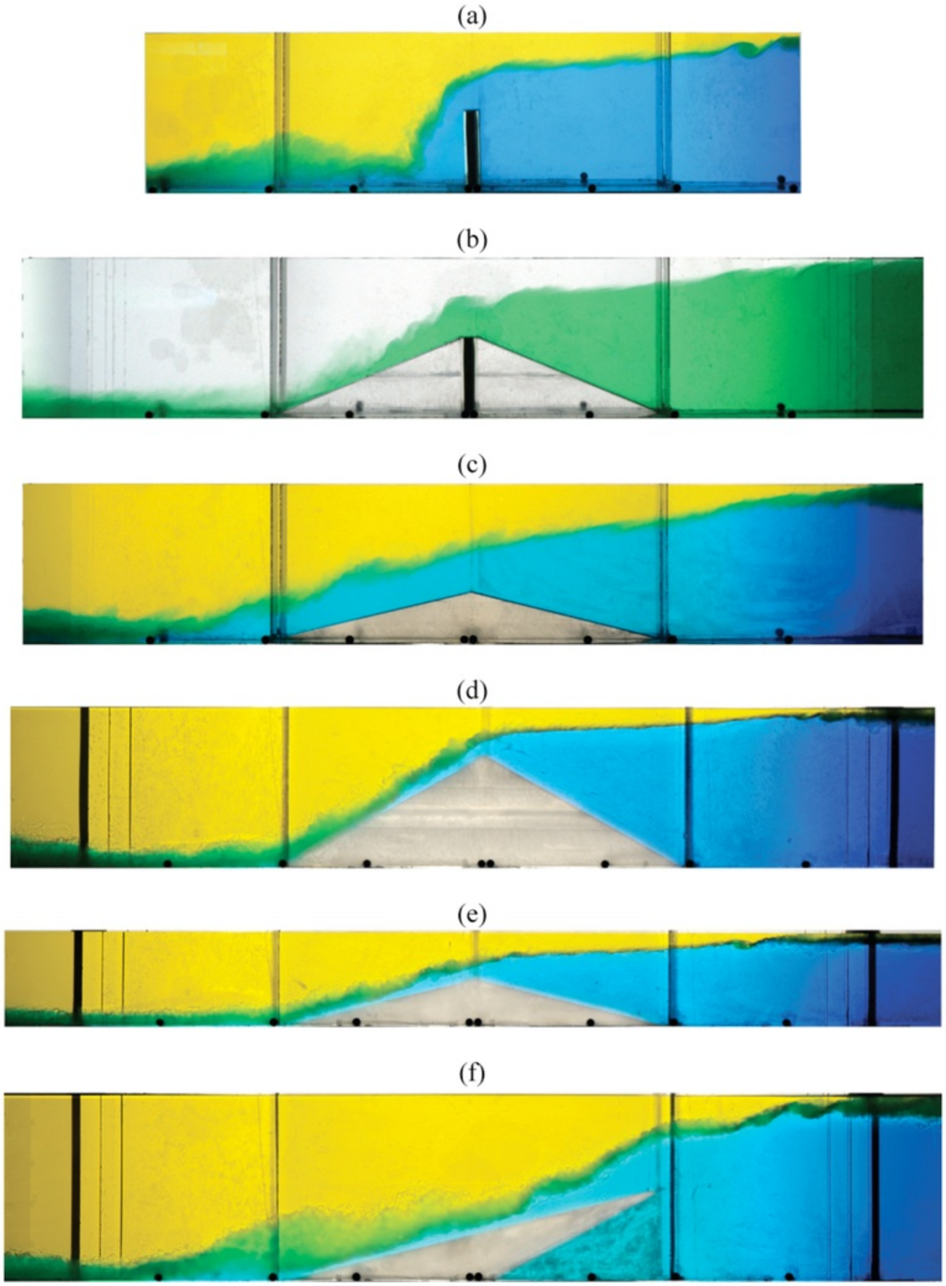

Figure 1. Photographs of the nominally steady exchange flow established shortly after withdrawal of the vertical barrier (which was initially situated at the apex of the sill in each case) in experiment: (a) J2; (b) K2; (c) L1; (d) L4; (e) L13; (f) L18. The reservoirs of light and dense water are on the left and right, respectively, of the sill. The vertical scale in each frame is proportional to the total depth H, and the constant width section of the channel in these cases is indicated by region lying between the first vertical grey bar on either side of the sill apex. The dimensionless density difference $\Delta\rho/\rho_0$ was approximately constant (i.e. 5%) in all cases shown.

A mixing efficiency η for the stratified turbulence in the flow can be calculated as

$$\eta = \Delta PE_m/\Delta APE. \quad (1)$$

This efficiency is equivalent to the ratio of time-integrated mixing and dissipation rates defined by Peltier & Caulfield (2003) for their numerical experiments.

3 OBSERVATIONS

In all cases, a qualitatively similar evolution ensued after withdrawal of the vertical barrier. Gravity currents of dense and light water propagated in opposite directions away from the sill apex leaving a nominally steady exchange flow with billows and intense turbulence on the density interface between the counter-flowing layers (Figure 1). The thickness of the dense overflow and the billow size is seen to scale with the depth of water above the sill apex. A clear asymmetry is also evident in the flow due to the bottom sill. Turbulence in the dense downslope flow develops to larger vertical scales than in the counter-flowing surface current, and usually extends through the overflow depth. Consequently, the dense current flowing to the left appears to carry little unmixed source water.

4 RESULTS

Prastowo et al. (2008, 2009) suggested that mixing behaviour is dependent upon a Reynolds number calculated using a velocity scale based on the buoyancy difference and the vertical size of the aperture through which exchange occurs, and a length scale based on the horizontal dimension L_c of the constant-width portion of the lateral constriction between reservoirs in which the sill resides. In the current configuration, this Reynolds number is expected to take the form

$$Re = 0.5[g'(H-h)]^{1/2}L_c/\nu, \quad (2)$$

where $g' = g\Delta\rho/\rho_0$ is the specific buoyancy anomaly between the reservoirs, g is the gravitational acceleration, ρ_0 is a reference density and ν is the kinematic viscosity of water. For a given sill/constriction geometry, the overall mixing efficiency is observed to reach an asymptote at sufficiently large Reynolds number. The maximum asymptotic values attained are 11% ± 1% and apply to exchange flows across a steep-sided sill (i.e. the "J" series weir) with $Re > 1 \times 10^5$. This maximum asymptotic value also coincides with that found for exchange flow in a simple lateral constriction by Prastowo et al. (2008). However, Figure 2 suggests that the large Reynolds number asymptote for mixing efficiency decreases as the dimensionless sill height increases and as the sill slope decreases. As the main interest in this paper is the role of sill geometry, we restrict our attention to experiments with sufficiently high Reynolds number to be in the asymptotic regime. Based on the data in Figure 2 we therefore consider only experiments for which $Re > 1 \times 10^5$ with a vertical weir and $Re > 0.5 \times 10^5$ with a triangular sill. All of the "L" series experiments satisfy the latter criteria.

The mixing efficiencies measured in the (large Reynolds number) experiments were smaller than the asymptotic maximum found by Prastowo et al. (2008, 2009), being dependent upon the sill slope h/L and relative sill depth h/H as suggested in Figure 2. However, neither of these dimensionless parameters yielded a satisfactory collapse of the data to a single curve. The main finding of this study is that a new dimensionless "slope length" parameter, $L/(H-h)$, instead appears to characterise the effect of sill geometry on mixing efficiency, as seen by the collapse of the data in Figure 3. In physical terms, we suggest that this parameter measures the length of density interface upon which shear instability is active, relative to the overflow current depth. When $L/(H-h)$ is small, mixing is likely to be most efficient because the billows do not extend through the current depth. However, as $L/(H-h)$ increases, the billows are constrained by the presence of the sill and mixing erodes the density anomaly in the current, as is apparent in Figure 1 for which the corresponding dimensionless slope parameters are (a) 0, (b) 2.5, (c) 1.79, (d) 4.17, (e) 4.17 and (f) 4.15.

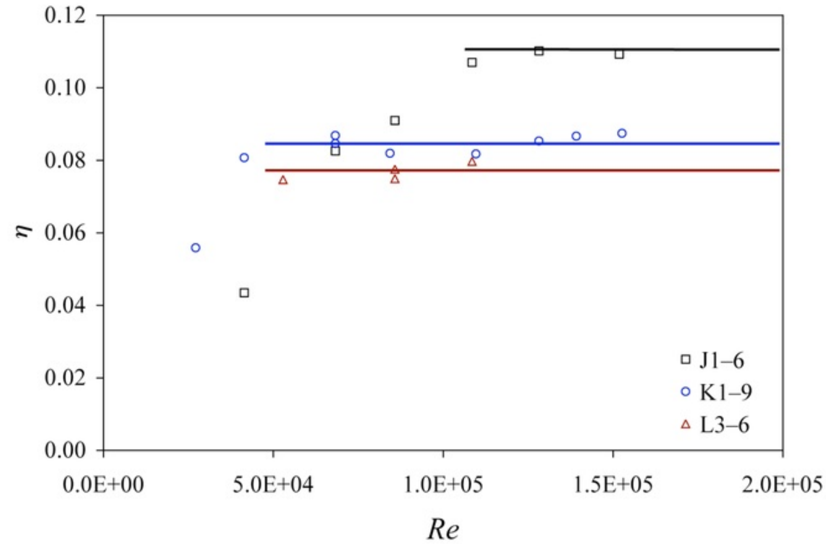

Figure 2: Mixing efficiency η as a function of Reynolds number *Re* for the subset of experiments in Table 1 in which $\Delta\rho/\rho_0$ was systematically varied. The uppermost solid line denotes the maximum asymptotic efficiency found in all the previous experiments reported by Prastowo et al. (2008, 2009), which include exchange flows through a contraction without a bottom sill. The uppermost line also approximately coincides with the maximum efficiency found for a steep-sided sill (i.e. J1–6, vertical weir) occupying half of the total water depth. The intermediate solid line indicates the asymptotic efficiency found for a triangular sill (i.e. K1–9, more gently sloping sides) of the same height. The lowermost solid line indicates the asymptotic efficiency found for a triangular sill of greater height (and hence greater slope, L3–6).

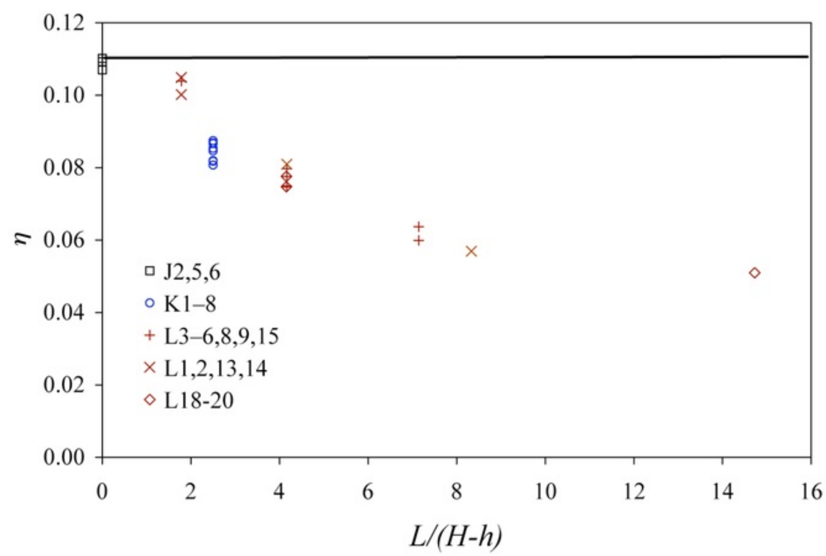

Figure 3: Mixing efficiency η as a function of the slope length parameter $L/(H-h)$ for the large Reynolds number subset of experiments in Table 1. The data points corresponding to triangular sills are grouped by sill height h, and the data from experiments with a vertical weir are plotted at $L/(H-h) = 0$. The solid horizontal line denotes the maximum asymptotic efficiency found in all the previous experiments reported by Prastowo et al. (2008, 2009), which include exchange flows through a contraction without a bottom sill.

5 CONCLUSIONS

All controlled exchange flows examined here are characterised by cumulative mixing efficiencies that are less than about 11% and are dependent upon sill geometry, but with little sensitivity to other variables including the driving buoyancy anomaly. The effect of sill geometry on the mixing appears to be captured by a single dimensionless parameter that is approximately proportional to the ratio of the slope length and the depth of the dense current overflow. The mixing efficiency is reduced as this ratio increases.

Mixing in exchange flows constitutes a fundamental process in the circulation of the global oceans, and the findings in this study have important consequences for the corresponding energy balances. In particular, turbulent mixing is commonly assumed to have an efficiency of 20% in oceanographic applications. However, we anticipate that the oceanic mixing associated with exchange flows could be substantially smaller.

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